SCATTERING OF ELECTRON BY EXCITED HELIUM ATOM

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ABSTRACT. Born's first approximation formula has been applied here to calculate the differential cross section of scattering of an electron by an excited helium atom. It is found that the cross section of scattering of electrons of 700 eV energy by excited He atom is very nearly the same as that by screened He atom ; the cross section of scattering of the same by bare He nucleus is slightly greater than both of them. However, the experimental cross section for ordinary He is considerably larger than these three theoretical results at angles above 80°, whereas at angles below 70°, the experimental values are less than all the theoretical values.

INTRODUCTION

Various models have been proposed to take into account the screening effect of the two electrons surrounding the nuclear charge. To calculate the above screening, Hylleraas (1929) has taken the wave function of the Schrödinger equation to be a product of two wave functions in the 1s state with $Z = \frac{27}{16}$, Z being the nuclear charge value.

Huzinaga (1960), on the other hand, has taken symmetrized product of two wave functionsa in 1s state with two different Z values.

In the present paper we propose to study the problem of scattering of electrons if the target atoms are already excited. In actual experimental conditions, the target atoms are also excited by inelastic collisions, so that there is always a certain fraction of the number of atoms which are in excited states ; it is worth while to see how the scattering is effected by the excited states of the target atoms. Here we are considering one of the electrons to be in its 1s state and the other in 2s state.

MATHEMATICAL RESULTS

The potential function of atom having two electrons is given by

$$V = -e^2 \int_{\tau_1=0}^{\infty} \int_{\tau_2=0}^{\infty} \psi^* \left\{ \frac{2}{r} - \frac{1}{|r-r_1|} - \frac{1}{|r-r_2|} \right\} \psi d\tau_1 \, d\tau_2 \qquad \dots (1)$$

where the co-ordinates of the two electrons of the atom are denoted by r_1 and

r_2 and r is the co-ordinate of the incident electron with the nucleus as the origin. ψ is the wave function of the system and is taken to be

$$\psi = \frac{N}{\sqrt{2}} \{R_1(r_1)\, R_2(r_2) + R_1\,(r_2)\, R_2(r_1)\}$$

where
$$R_1 = \left(\frac{Z_1}{a_0}\right)^{\frac{3}{2}} 2e^{-\frac{Z_1 r}{a_0}}$$

$$R_2 = \left(\frac{Z_2}{2a_0}\right)^{\frac{3}{2}} \left(2 - \frac{Z_2 r}{a_0}\right) e^{-\frac{Z_2 r}{a_0}}$$

and the normalization factor N is given by

$$N^2 = \frac{1}{4\pi^2 \left\{1 + 8Z_1^3 Z_2^3 \left(\frac{2}{2Z_1+Z_2}\right)^8 (Z_1+Z_2)^2\right\}}$$

Substituting (2) in (1) we obtain for V the following expression

$$V = -(4\pi)^2 N^2 e^2 \Bigg[\frac{2}{r} \left\{ 8Z_1^3 Z_2^3 \left(\frac{2}{2Z_1+Z_2}\right)^8 (Z_1 - Z_2)\right\}$$

$$+ \left(\frac{1}{r} + \frac{Z_1}{a_0}\right) e^{-\frac{2Z_1 r}{a_0}} + \left(\frac{1}{r} + \frac{3Z_2}{4a_0} + \frac{1}{4}\frac{Z_2^2}{a_0^2} r + \frac{1}{8}\frac{Z_2^3}{a_0^3} r^2\right) e^{-\frac{Z_2 r}{a_0}}$$

$$+ e^{-\frac{r}{2a_0}(2Z_1+Z_2)} \left\{\frac{4Z_1^3 Z_2^3}{a_0^3}\left(\frac{2}{2Z_1+Z_2}\right)^4 (Z_1 - Z_2)\right\} \left\{\frac{2(Z_1-Z_2)}{2Z_1+Z_2} r^2\right.$$

$$\left.- \frac{4Z_2 a_0 r}{(2Z_1+Z_2)^2} + \frac{8a_0^2(2Z_1 - 3Z_2)}{(2Z_1+Z_2)^3} - \frac{8(Z_1-Z_2)}{r}(2a_0)^3 \frac{1}{(2Z_1+Z_2)^4}\right\}\Bigg] \quad \ldots \quad (3)$$

To calculate the differential scattering cross section $\sigma(\theta)$ which is $|f(\theta)|^2$ we opply the Born approximation method in which $f(\theta)$ is given by

$$f(\theta) = -\frac{8\pi^2 m}{h^2} \int_0^\infty \frac{\sin kr}{kr} V(r)\, r^2\, dr \quad \ldots \quad (4)$$

Substituting the value of V from Eq. (3) in Eq. (4) we obtain

$$f(\theta)=\frac{2me^2}{\hbar^2k^2}\,\frac{1}{\left\{1+8Z_1^3Z_2^3\left(\frac{2}{2Z_1+Z_2}\right)^8(Z_1-Z_2)^2\right\}}$$

$$\times\left[16Z_1^3Z_2^3\left(\frac{2}{2Z_1+Z_2}\right)^8(Z_1-Z_2)^2\right.$$

$$+\frac{k^2a_0^2(k^2a_0^2+8Z_1^2}{(4Z_1^2+k^2a_0^2)^2}+\frac{k^2a_0^2}{(Z_2^2+k^2a_0^2)^4}\{7Z_2^6+4Z_2^4a_0^2k^2$$

$$\left.+4Z_2^2a_0^4k^4+a_0^6k^6\}+\frac{4Z_1^3Z_2^3}{a_0^3}\,\frac{\left(\frac{2}{2Z_1+Z_2}\right)^4(Z_1-Z_3)(2a_0)^4R}{(2Z_1+Z_2)^4\{(2Z_1+Z_2)^2+4k^2a_0^2\}}\right]\quad\ldots\quad(5)$$

where $R=4\{2(2Z_1+Z_2)^3ka_0-8(2Z_1+Z_2)k^3a_0^3\}\{64Z_1^4+16Z_1^3Z_2$

$$+120Z_1^2Z_2^2-76Z_1Z_2^3+14Z_2^4+32Z_1^3k^2a_0^2-8Z_2^2k^2a_0^2\}$$

$$+2ka_0(32Z_1^2+40Z_1^2Z_2+8Z_1Z_2^2)\{(2Z_1+Z_2)^4-24(Z_2+2Z_1)k^2a_0^2+16k^4a_0^4\}$$

If we put $Z_1=2$, $Z_2=1$, we get

$$f(\theta)=\frac{2a_0}{k^2a_0^2\left\{1+64\left(\frac{2}{5}\right)^8\right\}}\left[128\left(\frac{2}{5}\right)^8+\frac{k^2a_0^2(32+k^2a_0^2)}{(16+k^2a_0^2)^2}\right.$$

$$\left.+\frac{k^2a_0^2}{(1+k^2a_0^2)^4}\left\{(7+4k^2a_0^2+4k^4a_0^4+k^6a_0^6)\right\}\right]$$

In the above we have neglected the contribution of the last term of the expression (5) as it is very small compared with those of the other terms.

DISCUSSIONS

In the table below we have given the numerical values of the differential cross section at different scattering angles for the incident electron energy of 700 ev. For comparison we also give similar values of the differential cross section when the screening due to the electrons is completely neglected. In the third column the experimental values (Hughes, Mac Millan and Webb, 1932) are added.

TABLE I

Energy 700 ev

$|f(\theta)|^2$ in units of 10^{-20} cm^2

θ	$\lvert f(\theta)\rvert^2$ Present result	Coulomb field	Experimental values
57°	17.19	20.25	15.5
72°	8.4	8.8	8.12
87°	4.46	4.7	5.10
102°	2.37	2.88	3.97
117°	1.97	2.00	3.56
132°	1.48	1.53	3.44
147°	0.9	1.25	1.54

From the above table we find that the present theoretical cross section of scattering of electrons of 700 ev energy by excited He atoms is considerably

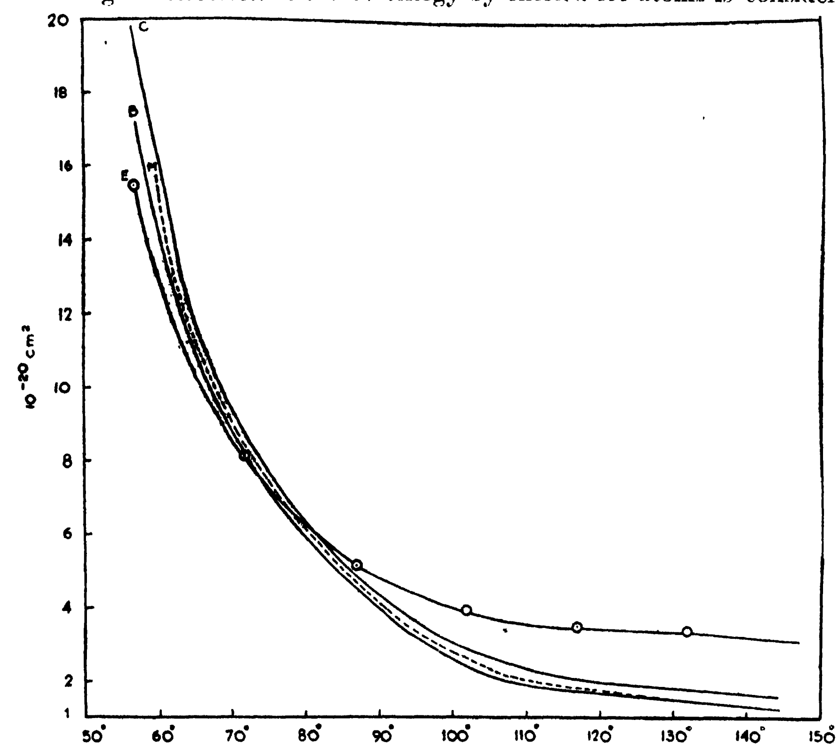

Fig. 1. Differential scattering cross section is plotted against angles in degrees. The curves marked E, B, M and C represent respectively the experimental results and the theoretical results of Bhattacharyya, Mukherji and of coulomb scattering of the bare nucleus.

lower than the experimental cross section of scattering by ordinary He, at large angles of scattering. The theoretical cross section of scattering by screened He atoms in the ground state calculated by the method of Born's first approximation (Mukherjee, 1961) is given in the graph and found to be very nearly the same as the scattering cross section by excited He atoms at angles above 90°, and slightly larger at angle below 90°. The cross section of scattering by bare nucleus calculated by the first Born approximation method is larger than both the theoretical cross sections either by ordinary He atoms or by excited He atoms; this shows that the influence of screening is not so appreciable at this energy of the incident electron. Moreover, the fact that the cross section of scattering even by bare nucleus calculated in the first Born approximation is much lower than the experimental findings at angles greater than 80°, seems to indicate the inadequacy of the first Born approximation and the necessity of taking into account higher order terms of Born series.

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